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Formation of massive stars and black holes in self-gravitating AGN discs, and gravitational waves in LISA band.

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ABSTRACT

We propose a scenario in which massive stars form at the outer edges of an AGN accretion disc. We analyze the dynamics of a disc forming around a supermassive black hole, in which the angular momentum is transported by turbulence induced by the disc's self-gravity. We find that once the surface density of the disc exceeds a critical value, the disc fragments into dense clumps. We argue that the clumps accrete material from the remaining disc and merge into larger clumps; the upper mass of a merged clump is a few tens to a few hundreds of solar mass. The biggest clumps collapse and form massive stars, which produce few-tens-solar-mass black holes at the end of their evolution.

We construct a model of the AGN disc which includes extra heat sources from the embedded black holes. If the embedded black holes can accrete at the Bondi rate, then the feedback from accretion onto the embedded black holes may stabilize the AGN disc against the Toomre instability for an interesting range of the model parameters. By contrast, if the rate of accretion onto the embedded holes is below the Eddington limit, then the extra heating is insufficient to stabilize the disc.

The embedded black holes will interact gravitationally with the massive accretion disc and be dragged towards the central black hole. Merger of a disc-born black hole with the central black hole will produce a burst of gravitational waves. If the central black hole is accreting at a rate comparable to the Eddington limit, the gas drag from the accretion disc will not alter significantly the dynamics of the final year of merger, and the gravitational waves should be observable by LISA. We argue that for a reasonable range of parameters such mergers will be detected monthly, and that the gravitational-wave signal from these mergers is distinct from that of other merger scenarios. Also, for some plausible black hole masses and accretion rates, the burst of gravitational waves should be accompanied by a detectable change in the optical luminosity of the central engine.

1 INTRODUCTION

It is widely believed that self-gravitating accretion discs can form around supermassive black holes in AGNs. Theoretical models show that the AGN accretion discs must become self-gravitating if they extend beyond a fraction of a parsec away from a central black hole (Paczynski 1978, Kolykhalov and Sunyaev 1980, Schlosman and Begelman 1987, Kumar 1999, Jure 1999, Goodman 2002). Self-gravitating discs are unstable to fragmentation on a dynamical timescale; self-gravity of AGN accretion discs is a major issue in understanding how gas is delivered to the central black hole. It is likely that star formation will occur in the outer parts of an AGN accretion disc (Kolykhalov and Sunyaev 1980, Schlosman and Begelman 1987).

Recently, some supporting evidence for this process was obtained from observations of stellar velocities in our galactic center. Levin and Beloborodov (2003, LB) have analyzed the 3-D velocity data of Genzel et. al. (2000), and have found that ~ 80 percent of massive stars from the sample move in a thin disc around the SgrA* black hole. LB's finding was confirmed by Genzel et. al. (2003), who have used the most recent data from Ott et. al. (2003). Genzel et. al. (2003) have also found evidence for a second disc of stars which has a larger radius and thickness than the disc in LB; the relative orientation of the two discs is nearly orthogonal. LB have argued that the stellar disc is a remant of a dense self-gravitating accretion disc, much like what was already proposed by Kolykhalov and Sunyaev in 1980 for the circum-black-hole accretion discs in other galaxies (we quote from Kolykhalov and Sunyaev's abstract: "The fragmentation would produce around the black hole a ring of gas and stars which would survive even after accretion onto the hole

has ceased".) The idea for star formation in the disc around SgrA* was originally proposed by Morris et. al. (1999); in their scenario the disc's self-gravity does not play a central role. Instead, the compression of the disc material is achieved by radiation and wind pressure from the central engine. Morris et. al. (1999) have identified the circumnuclear ring as the most likely source of supply for the disc material. However, the circumnuclear ring is strongly misaligned with the disc found by LB and by Genzel et. al. (2003). Thus, it seems more likely that the gas in the disc would come from a disrupted molecular cloud on a plunging trajectory (Sanders 1998).

A disc of stars could also be the remnant of a tidally disrupted young stellar cluster (Gerhard 2001, Portegiese Zwart, McMillan, and Gerhard 2003, McMillan and Portegiese Zwart 2003). This scenario becomes more credible if the dense cluster core contains an intermediate-mass black hole (Hansen and Milosavlevic, 2003). The gravitational pull from the latter would allow the core to stay intact until it comes within 0.1pc from the SgrA*, where the massive stars are observed to reside.

Both the disc fragmentation and disrupted cluster scenarios have implications for the dynamics of the galactic nuclei and may provide channels for producing bursts of gravitational waves detectable by LISA. In this paper we investigate the consequences of disc fragmentation.

The dynamics of the fragmenting disc is strongly affected by the feedback energy input from the starburst. Colliln and Zahn (1999) have conjectured that the feedback from this star formation will prevent the accretion disc from becoming strongly self-gravitating. However, Goodman (2003) has used general energy arguments to show that the feedback from star formation is insufficient to prevent an AGN disc with the near-Eddington accretion rate from becoming strongly self-gravitating at a distance of ∼pc from the central black hole. This is distinct from the case of galactic gas discs, for which there is evidence that the feedback from star formation protects them from their self-gravitaty.

In this paper we concentrate on the physics of the self-gravitating disc and make a semi-analytical estimate of the possible mass range of stars formed in such discs (Sections II and III). Our principal conclusion is that the stars can be very massive, up to hundreds of solar masses. These massive stars evolve quickly and produce stellar-mass black holes as the end product of their evolution.

Goodman (2003) has mentioned the possibility that the AGN disc is stabilized by feedback from accretion onto black holes embedded in the disc[⋆]. In Section IV, we investigate Goodman's conjecture and study whether the feedback from accretion onto these black holes may indeed stabilize a disc around a supermassive black hole, thus helping to fuel bright AGNs. We find that if the embedded black holes are accreting at a super-Eddington rate set by the Bondi-Hoyle formula, then the extra heating they provide may be able to stabilize the AGN disc. However, if their acretion rate is no higher than the Eddington limit, than the AGN disc cannot be stabilized against its own selfgravity, and one would need some other source of heating to make the disc stable.

We then argue (Section V) that the disc-born black hole interacts gravitationally with the accretion disc, and migrates inward on the timescale of $\sim 10^7$ years. The merger of the migrating black hole with the central black hole will produce gravitational waves. We show that for a broad range of AGN accretion rates the final inspiral is unaffected by gas drag, and therefore the gravitational-wave signal should be detectable by LISA. The rate of these mergers is uncertain, but if even a fraction of a percent of the disc mass is converted into black holes which later merge with the central black hole, then LISA should detect monthly a signal from such a merger. The final inspiral may occur close to the equatorial plane of the central supermassive hole and is likely to follow a quasi-circular orbit, which would make the gravitational-wave signal distinct from those in other astrophysical merger scenarios. If the disc-born black hole is sufficiently massive, it will disrupt accretion flow in the disc during the final year of its inpiral, thus making an optical counterpart to the gravitational-wave signal.

2 PHYSICS OF AN ACCRETING SELF-GRAVITATING DISC

The importance of self-gravitating accretion discs in astrophysics has long been understood (Pazynsky 1978, Lin and Pringle 1987). It was conjectured that the turbulence generated by the gravitational (Toomre) instability may act as a source of viscosity in the disc. This viscosity would both drive accretion and keep the disc hot; the latter would act as a negative feedback for the Toomre instability and would keep the disc only marginally unstable. Recently, there has been big progress in our understanding of the self-gravitating discs, due to a range of new and sophisticated numerical simulations (Gammie 2001, Mayer et. al. 2002, Rice et. al. 2003). In our analysis, we shall rely extensively on these recent numerical results.

Consider an accretion disc which is supplied by a gas infall. This situation may arise when a merger or some other major event in a galaxy delivers gas to the proximity of a supermassive black hole residing in the galactic bulge. Let $\Sigma(r)$ be the surface density of the disc. We follow the evolution of the disc as $\Sigma(r)$ gradually increases due to the infall.

We begin by assuming that initially there is no viscosity mechanism, like Magneto-Rotational Instability (MRI), to transport the angular momentum and keep the disc hot[†]. This assumption is valid when the ionization fraction of the gas in the disc is low, i.e. when the gas is far enough from a central source (about a thousand Schwarzchild radii from the supermassive black hole). We also, for the time being, neglect irradiation from the central source; this may be a good assumption if a disrupted molecular cloud forms a disc but the accretion onto the hole has not yet begun. As will

⋆ Previously, Kolykhalov and Sunyaev (1980) have suggested that the x-rays produced by accretion onto the embedded black holes would be absorbed by the disc and reradiated in the infrared. The luminocity and spectra of the infrared radiation of the internally heated AGN disc was recently computed by Sirko and Goodman (2003).

† When the disc begins to fragment, the viscousity due to self-gravity-driven shocks exceeds the one due to MRI; see below. Therefore, while computing the disc parameters at fragmentation, it is reasonable to ignore MRI

be discussed below, irradiation is important for some regions in the AGN discs we are considering. However, as shown in the following subsection, inclusion of irradiation or other source of heating will only strengthen the case for formation of massive stars.

We assume, therefore, that the forming disc cools until it becomes self-gravitating; this happens when

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \simeq 1. \quad (1)$$

Here c_s is the isothermal speed of sound at the midplane of the disc, and Ω is the angular velocity of the disc. Numerical simulations show that once the disc becomes self-gravitating, turbulence and shocks develop; they transport angular momentum and provide heating which compensates the cooling of the disc (Gammie 2001). We thus assume, in agreement with the simulations, that when the disc exists, it is marginally self-gravitating, i.e. $Q = 1$. Then

$$c_s = \frac{\pi G \Sigma}{\Omega}, \quad (2)$$

and

$$T \sim 2m_p c_s^2/k_B = \frac{2m_p}{k_B}\left(\frac{\pi G \Sigma}{\Omega}\right)^2. \quad (3)$$

Here T is the temperature in the midplane of the disc, m_p is the proton mass, and k_B is the Boltzmann constant.

The one-sided flux from the disc surface is given by the modified Stephan-Boltzman law:

$$F = \sigma T_{\rm eff}^4, \quad (4)$$

where $T_{\rm eff}$ is the effective temperature. It is related to the midplane temperature by

$$T_{\rm eff}^4 \sim T^4 f(\tau) = f(\tau)\left(\frac{2m_p}{k_B}\right)^2 \left(\frac{\pi G \Sigma}{\Omega}\right)^8 ., \quad (5)$$

where $\tau = \kappa\Sigma/2$ is the optical depth of the disc; here $\kappa(T)$ is the opacity of the disc. The function $f(\tau) = \tau$ for an optically thin disc, and $f(\tau) = 1/\tau$ for an optically thick disc. We combine these two cases in our model by taking

$$f(\tau) = \frac{\tau}{\tau^2 + 1}. \quad (6)$$

We have used Eq. (3) in the last step of Eq. (5). The flux from the disc is powered by the accretion energy:

$$F = \frac{3}{8\pi}\Omega^2 \dot{M} = \frac{9}{8}\alpha\Omega c_s^2 \Sigma = \frac{9}{8}\alpha(\pi G)^2 \frac{\Sigma^3}{\Omega}. \quad (7)$$

Here α parametrizes viscous dissipation due to self-gravity (Gammie 2001); we have used Eq. (2) in the last step. Using Eqs (4), (7), and (5), we can express the viscosity parameter α as

$$\alpha = \frac{8\sigma}{9}\left(\frac{2m_p}{k_B}\right)^4 (\pi G)^6 f(\tau) \frac{\Sigma^5}{\Omega^7}. \quad (8)$$

It is very important to emphasize that in this model α is only a function of Σ and Ω: the temperature in the midplane is determined by Eq. (3), and this temperature sets the opacity which in turn determines the optical depth $\tau = \kappa\Sigma/2$. The opacity in the range of temperatures and densities of interest to us is set by light scattering off ice grains and, in some cases, by scattering off metal dust. The relevant regimes are worked out in the literature on protoplanetary discs; we use the analytical fit to the opacity (in cm^2/gm) from the appendix of Bell and Lin, 1994.

$$\begin{aligned} \kappa &= 0.0002 \times T_K^2 && \text{for} && T < 166\text{K}, \\ \kappa &= 2 \times 10^{-16} T_K^{-7} && \text{for} && 166\text{K} < T < 202\text{K} \\ \kappa &= 0.1\sqrt{T_K} && \text{for} && T > 202\text{K}. \end{aligned} \quad (9)$$

In the first interval the opacity is due to icegrains; in the second interval the icegrains evaporate, and the opacity drops sharply with the temperature; in the third interval (the highest T) dust grains are the major source of the opacity. Levin and Matzner (2003, in preparation) consider a more general temperature regime and use the available opacity tables instead of relying on the analytical fits.

From Eq. (8) we see that as Σ of the disc increases due to the merger-driven infall, the effective viscosity will reach $\alpha_{\rm crit} \sim 1$. At this stage the cooling time of the disc becomes comparable to the orbital period. Gammie's simulations show that in this case the turbulence induced by self-gravity is no longer able to keep the disc together, and the disc fragments. Gammie's simulations give $\alpha_{\rm crit} \simeq 0.3$; the numerical value we quote disagrees with Gammie's, but agrees with $\alpha_{\rm crit}$ quoted by Goodman (2003) since like Goodman we use isothermal speed of sound for α-prescription. Gammie's results, although obtained for razor-thin discs, justify the key assumption of our model: *the disc exists as a whole for* $\alpha < \alpha_{\rm crit}$, *and fragments once* $\alpha = \alpha_{\rm crit}$. Similar criterion for the disc fragmentation was already used in Shlosman and Begelman, 1987.

We shall refer to the midplane temperature and surface density of the marginally fragmenting disc with $\alpha = \alpha_{\rm crit}$ as the critical temperature $T_{\rm crit}$ and the critical surface density, $\Sigma_{\rm crit}$.

We now find the critical surface density and midplane temperature as a function of Ω. We use Eq. (3) to express Σ as a function of T and Ω, then substitute this function into Eqs. (6) and (8), and set $\alpha = \alpha_{\rm crit}$. After simple algebra, we obtain

$$\Omega^3 + p\Omega = q, \quad (10)$$

where

$$\begin{aligned} p &= 2\left(\frac{\pi G}{\kappa(T_{\rm crit})}\right)^2 \frac{m_p}{k_B T_{\rm crit}}, \\ q &= \frac{32\sigma}{9\alpha_{\rm crit}} \frac{(\pi G)^2}{\kappa(T_{\rm crit})} (m_p/k_B)^2 T_{\rm crit}^2. \end{aligned} \quad (11)$$

There is an analytical solution to Eq. (10):

$$\Omega = w - p/(3w), \quad (12)$$

where

$$w = \{q/2 + [(q/2)^2 + (p/3)^3]^{1/2}\}^{1/3}. \quad (13)$$

In Figure 1 we make a plot of $T_{\rm crit}$ as a function of the orbital period, for concreteness we set $\alpha_{\rm crit} = 0.3$. The critically self-gravitating disc is optically thin if the second term of the LHS of Eq. (10) is dominant, and optically thick otherwise. This can be expressed as a condition on the critical temperature: the disc is optically thin if

$$T_{\rm crit} < 14\text{K}\alpha_{\rm crit}^{2/15}, \quad (14)$$

and optically thick for higher critical temperatures. The an-

gular frequency above which the critically unfragmented disc becomes optically thick is

$$\Omega_{\rm transition} \simeq 16.3 \times 10^{-11} {\rm sec}^{-1}. \tag{15}$$

We use Eqs. (2) and (3) to find the critical surface density $\Sigma_{\rm crit}$, which is plotted in Fig. 2, and the scaleheight $h_{\rm crit} = c_s/\Omega$ of a marginally fragmenting disc. The Toomre mass $\bar{M}_{\rm cl} = \Sigma_{\rm crit} h_{\rm crit}^2$ is the mass scale of the first clumps which form in the first stage of fragmentation. In Fig. 3, we plot the Toomre mass of the critically fragmenting disc as a function of the orbital period.

The value of $\bar{M}_{\rm cl}$ is not large enough for the initial clump to open a gap in the accretion disc. The newly-born clump will therefore accrete from the disc. The Bondi-Hoyle estimate of the accretion rate gives $\dot{M}_{\rm cl} \sim \Omega \bar{M}_{\rm cl}$, i.e. we expect the mass of the new clump to grow on the dynamical timescale until it becomes large enough to open a gap in the gas disc. The upper limit $\tilde{M}_{\rm cl}$ of this value is the mass which opens a gap in the original gas disc with $\Sigma = \Sigma_{\rm crit}$ just before it fragments:

$$\tilde{M}_{\rm cl} \simeq \bar{M}_{\rm cl} \left(40\pi\alpha_{\rm crit}\right)^{1/2} \left(r/h_{\rm crit}\right)^{1/2}; \tag{16}$$

see Eq. (4) of Lin and Papaloizou (1986). Once the gas is depleted from the disc, we expect the initial distribution of the clump masses to be concentrated between $\bar{M}_{\rm cl}$ and $\tilde{M}_{\rm cl}$. The clump masses will evolve when clumps begin to merge with each other; this is addressed in section III.

2.0.0.1 Effect of irradiation and other sources of heating . So far in determining the structure of the self-gravitating disc, we have neglected external or internal heating of the disc. This is certainly a poor approximation in many cases. Irradiation from AGN or surrounding stars, or feedback from star formation inside the disc can be the dominant source of heating of the outer parts of AGN discs (eg, Shlosman and Begelman 1987, 1989). For example, iradiation from circumnuclear stars will keep the disc temperature at a few tens of Kelvin, which is higher than the critical temperature we obtained for a self-gravitating disc beyond 0.1pc away from $10^7 M_\odot$ black hole.

However, extra heating will always work to increase the critical temperature at which the disc fragmentation occurs. Therefore, the values of the critical surface density $\Sigma_{\rm cr}$, scaleheight $h_{\rm cr}$, and the mass of the initial fragment $M_{\rm cl}$ obtained above should be treated as lower bounds of what might be expected around real AGNs or protostars. Higher values of these quantities would only strengthen main conclusions of this paper. LB have found that when the rate of accretion $\dot{M}$ is constant throughout the disc, the Toomre mass $\bar{M}_{\rm cl}$ is given by

$$\bar{M}_{\rm cl} = 1.8 M_\odot \left(\frac{\alpha}{0.3}\right)^{-1} \times \frac{\dot{M}c^2}{L_{\rm edd}} \left(\frac{M}{3\times 10^6 M_\odot}\right)^{0.5} \left(\frac{r}{0.2{\rm pc}}\right)^{1.5}, \tag{17}$$

and that the gap-opening mass is

$$M_{\rm gap} = 62 M_\odot \left(\frac{\alpha}{0.3}\right)^{-0.5} \times \frac{\dot{M}c^2}{L_{\rm edd}} \left(\frac{M}{3\times 10^6 M_\odot}\right)^{0.5} \left(\frac{r}{0.2{\rm pc}}\right)^{1.5} \left(\frac{r}{10h}\right)^{0.5}. \tag{18}$$

Here, M is the black-hole mass, and r and h are the radius and the scaleheight of the disc. Thus, even if the clumps do not merge with each other and stop their growth at the gap-opening mass, the mass of the formed stars will be biased towards the high-mass end. In the next section we discuss the effects of clump mergers and the mass range of stars born after the disc fragments.

3 EVOLUTION OF THE FRAGMENTED DISC

Gammie's simulations show that once the disc fragments, the clumps merge and form significantly larger objects. In fact, his razor-thin shearing box turned into a single gas lump at the end of his simulation.

For merger to be possible, the clumps should not collapse into individual stars before they can coalesce with each other. Let's check if this is the case.

Consider a spherical nonrotating clump of radius R and mass $M_{\rm cl}$. First, assume that the clump is optically thin. The energy radiated from the clump per unit time is

$$W_{\rm cool} \sim \sigma T^4 R^2 \kappa(T) \Sigma \sim M_{\rm cl} \sigma T^4 \kappa(T). \tag{19}$$

This radiated power cannot exceed the clumps gravitational binding energy released in free-fall time, $G^{1.5} M^{2.5} R^{-2.5}$. Together with $\kappa(T) \propto T^2$ [since icegrains dominate opacity for the optically-thin marginally fragmenting disc–see Eq. (14)], this condition implies that

$$T < \tilde{T} = T_0 R^{-5/12}, \tag{20}$$

where T_0 is a constant for the collapsing optically thin clump. The temperature $\tilde{T}$ in Eq. (20) is less than the virial temperature, which scales as R^{-1}. Therefore, after the collapse commenses, the clump is not virialized while it is optically thin. The temperature cannot be much smaller than $\tilde{T}$, since otherwise the cooling rate would be much smaller than the rate of release of the gravitational binding energy, and the gas would heat up by quasi-adiabatic compression. The inequality in Eq. (20) should be substituted by an approximate equality, and therefore we have during optically-thin collapse

$$T \propto R^{-5/12}. \tag{21}$$

The optical depth scales as

$$\tau \propto R^{-11/3}. \tag{22}$$

and hence rises sharply as the clump's radius decreases; as the clump shrinks it becomes optically thick‡. It is possible to show that once the clump is optically thick, it virializes quickly with it's temperature $T \propto R^{-1}$. For $\kappa \propto T^2$ (icegrains), the cooling time of an optically thick clump scales with the clump radius as

$$t_{\rm cool} \propto R^{-3}. \tag{23}$$

‡ The contraction of an optically thin clump may be complicated by subfragmentation, since the Jean's mass for such clump scales as $R^{7/8} \propto \tau^{-0.23}$. We suspect that in most cases the clump becomes optically thick before it subfragments, since the Jean's mass has a slow dependence on the optical depth. However, only detailed numerical simulations can resolve these issues.

The characteristic timescale for the clump to collide with another clump scales with the clump radius as

$$t_{\rm collision} \propto R^{-2}. \tag{24}$$

From Eqs. (23) and (24), we see that the collision rate decreases less steeply than the coolling rate as a function of the radius of an optically thick clump. Therefore, merger can be an efficient way of increasing the clump's mass.

This conclusion is no longer valid when the temperature of the clump becomes larger than $\sim$ 200K; then the opacity is dominated by metal dust with $\kappa \propto T^{1/2}$. In this case the cooling time scales as $t_{\rm cool} \propto R^{-1.5}$. The collision timescale increases faster than the cooling time as the clump shrinks, and naively one would expect that mergers may not be efficient in growing the clump masses.

However, we have neglected the rotational support within a clump. Each clump is initially rotating with angular frequency comparable to the clump's inverse dynamical timescale; for example, in a Keplerian disc each clump's initial angular velocity is $\sim \Omega/2$. Therefore each clump will shrink and collapse into a rotationally supported disc, and the size of this disc is comparable to the size of the original clump (this picture seems to be in agreement with Gammie's simulations). Thus rotational support generally slows down the collapse of an individual fragment and makes mergers between diferent fragments to be efficient.

Magnetic braking is one of the ways for the clump to lose its rotational support[§] (see, e.g., Spitzer 1978). One generally expects a horizontal magnetic field to be present in a differentially rotating disc due to the MRI (Balbus and Hawley, 1991). Ionization fraction in the disc is expected to be small, so the magnetic field is saturated at a subequipartition value $B = \beta B_{\rm eq}$, with $\beta << 1$. Horizontal magnetic field will couple inner and outer parts of the differentially rotating clump on the Alfven crossing timescale $t_{\rm alfven} \sim t_{\rm dynamical}/\beta$, and the collapse will proceed on this timescale as well.

What is the maximum mass that the clump can achieve? This issue has been analyzed for the similar situation of a protoplanetary core accreting from a disc of planetesimals (Rafikov 2001 and references therein). The growing clump cannot accrete more mass than is present in it's "feeding annulus". This gives the maximum "isolation" mass of a clump:

$$M_{\rm is} \sim \frac{(2\pi r^2 \Sigma_{\rm crit})^{3/2}}{M^{1/2}} = 2\pi\sqrt{2}\bar{M}_{\rm cl}(r/h_{\rm crit})^{3/2}, \tag{25}$$

where, as above, $\bar{M}_{\rm cl} = \Sigma_{\rm crit} h_{\rm crit}^2$ is the mass scale of the first clumps to form from a disc; see, e.g., Eq. (2) of Rafikov (2001). However, numerical work of Ida and Makino (1993) and analytical calculations of Rafikov (2001) indicate that the isolation mass may be hard to reach. The consider a massive body moving on a circular orbit through a disc of gravitationally interacting particles, and they find that when the mass of the body exceeds some critical value, an annular gap is opened in the particle disc around the body's orbit. We can idealize a disc consisting of fragments as a disc of particles of a typical fragment mass $M_{\rm fr}$. Once a growing clump opens a gap in a disc of gravitationally interacting fragments, the clump's growth may become quenched. This gap-opening mass of the clump $M_{\rm gap}$ is given by Eq. (25) of Rafikov (2001):

$$\frac{M_{\rm gap}}{M_{\rm is}} = \frac{I}{2^{7/6}\pi^{1/2}} \left(\frac{M_{\rm fr}}{\Sigma r^2}\right)^{1/3} \left(\frac{M}{\Sigma r^2}\right)^{1/2}. \tag{26}$$

We use the numerical factor $2^{-7/6}\pi^{-1/2}I = 1.5$ appropriate for thin discs. By taking $Q = 1$ we get

$$M_{\rm gap} \simeq 14 M_{\rm fr}(M_{\rm fr}/\bar{M}_{\rm cl})^{1/3}(r/h_{\rm crit}). \tag{27}$$

In Figure 4 the masses $M_{\rm is}$ and $M_{\rm gap}$ are plotted as a function of radius for a $3\times10^6 M_\odot$ black hole; when we calculate $M_{\rm gap}$ we conservatively set $M_{\rm fr} = \bar{M}_{\rm cl}$ and not to the larger value $\tilde{M}_{\rm cl}$. It is likely that the most massive clumps will reach $M_{\rm gap}$, but it will be more difficult to form a clump with the mass $M_{\rm is}$.

From Fig. 4 we see that the most massive clumps can reach tens hundereds of solar masses. The maximum mass would be even larger if we included the heating of the disc by external irradiation or internal starburst. It is plausible that these very massive clumps will form massive stars; the masses of the stars may be comparable to the masses of the original clumps; see McKee and Tan (2002) and references therein. Stars with masses of a few tens of solar masses will produce black holes as the end product of their rapid ($< 10^6$yr) evolution; the characteristic mass of these black holes is believed to be around $10M_\odot$. A recent work by Mirabel and Rodrigues (2003) shows that the black holes whose projenitors have masses $> 40M_\odot$ do not recieve a velocity kick at their birth. Thus, the disc-born black holes are likely to remain embedded in the disc.

4 AGN DISC WITH EMBEDDED STELLAR-MASS BLACK HOLES

Once formed, the stellar-mass black holes will dramatically affect the dynamics of the AGN disc. We assume that the disc is replenished due to continuous infall, and in this section we analyze a steady-state AGN disc which is heated by the energy released due to the the accretion onto stellar-mass black holes embedded in the disc.

We make an anzatz that the heated disc is optically thick and is supported by the radiation pressure. Once we obtain the solution for the structure of the disc, we will derive the regime of validity for our anzats. We thus have for the sound speed in the disc midplane:

$$c_s^2 = \frac{aT^4}{3\rho} \sim \frac{2aT^4}{3\Sigma}\frac{c_s}{\Omega}, \tag{28}$$

and hence

$$c_s \sim \frac{4\kappa F}{3\Omega c}. \tag{29}$$

Here we have used the expression for the flux through the disc face, $F \sim caT^4/(2\Sigma\kappa)$.

For simplicity, we assume that there are $N_{\rm bh}$ black holes of mass $M_{\rm bh}$ embedded in a disc within radius r from the central hole of mass M. For this "planetary" system to be stable, $N_{\rm bh}$ cannot be much bigger than $(M/M_{\rm bh})^{1/3}$, i.e. the embedded holes cannot be within each other's Hill spheres.

To make further progress, we need to know the rate of accretion onto the black holes embedded in the disc. Because the density of the disc material is high, the well-known

§ Another way is via collisions with other clumps.

Bondi-Hoyle formula gives a super-Eddington accretion rate; it is not clear at this moment how feasible this is (but see e.g. Begelman 2002). We therefore consider below two separate models: the "Bondi-Hoyle" model, in which the embedded black holes accrete at the Bondi-Hoyle rate, and the "Eddington" model, in which the accretion onto the embedded holes occurs at the Eddington limit.

4.1 The "Bondi-Hoyle" model

. In this model, the accretion rate for the embedded black home of mass $M_{\rm bh}$ is given by the Bondi-Hoyle formula,

$$\dot{M}_{\rm bh} \simeq 4\pi r_{\rm bondi}^2 \rho c_s \sim 2\pi r_{\rm bondi}^2 \Sigma\Omega. \tag{30}$$

For the latter expression to be true, the Bondi radius $r_{\rm bondi} = GM_{\rm bh}/c_s^2$ must be less than the scaleheight of the disc; this condition will be checked once the structure of the disc is worked out.

We assume that the fraction ϵ of the rest-mass energy of the accreted material goes into heating of the disc¶, is thermalized, and is eventually radiated as flux F from the disc surface:

$$F = \epsilon \frac{N_{\rm bh}}{2\pi r^2} \dot{M}_{\rm bh} c^2. \tag{31}$$

Finally, the equation for the central hole's accretion rate

$$\dot{M} = 3\pi\alpha\Sigma c_s^2/\Omega, \tag{32}$$

together with the Equations (29), (30), and (31) allow us to find the structure of the disc once the quantities $M_{\rm bh}$, $N_{\rm bh}$, M, $\dot{M} = \dot{m}\dot{M}_{\rm edd}$, r, α, κ, and ϵ are fixed. After some algebra, we find the expression for the Toomre parameter of the disc

$$Q \sim 10 \frac{\alpha_{0.3}^{4/7} (\kappa/\kappa_0)(\epsilon/\dot{m})^{4/7} \, (M_{\rm bh}/10M_\odot)^{5/7}}{M_7^{3/14} (r/0.1{\rm pc})^{3/2}}. \tag{33}$$

We see that a typical AGN disc can be stabilized by feedback from embedded black holes out to a radius of about a parsec. Here κ_0 is the opacity for Thompson scattering, and $N_{\rm bh}$ was set to its upper limit, $(M/M_{\rm bh})^{1/3}$.

Recently, Sirko and Goodman (2002) analyzed SEDs from AGN discs in which $Q = 1$ is enforced by the unspecified heating sources. They have shown that current observations limit such self-gravitating discs to be no larger than $\sim 10^5$ Schwartzchild radii, about 0.1pc for $10^7 M_{\rm odot}$ black hole. This motivates our normalization for the radius used in Eq. (33).

We now check the assumptions which were used in calculating the disc structure:

$$\frac{p_{\rm rad}}{p_{\rm gas}} \sim 1.5 * 10^2 \frac{(\kappa/\kappa_0)^{1/4} M_7^{5/56} (M_{\rm bh}/10M_\odot)^{15/28}}{\alpha_{0.3}^{1/14} (\epsilon/\dot{m})^{1/14} (r/0.1{\rm pc})^{3/8}} \gg 1, \tag{34}$$

so the disc is radiation-pressure dominated;

$$\tau \sim 10^2 \frac{(\dot{m}/\epsilon)^{5/7} M_7^{41/42}}{(M_{\rm bh}/10M_{\rm odot})^{10/21} (r/0.1{\rm pc})^{1/2} \alpha_{0.3}^{5/7}} \gg 1, \tag{35}$$

hence the disc is optically thick; and

$$r_{\rm bondi}/h = 2 * 10^{-3} \frac{\alpha_{0.3}^{3/7} \epsilon^{3/7} (M_{\rm bh}/10M_\odot)^{2/7}}{M_7^{2/7} \dot{m}^{3/7}} \ll 1, \tag{36}$$

which gives some credibility to the Bondi-Hoyle estimate of the accretion rate onto the disc-born black hole.

The expressions for other useful quantities characterizing the disc are given below:

$$h/r \sim 0.08 \frac{(M_{\rm bh}/10M_\odot)^{5/21} (\dot{m}/\epsilon)^{1/7}}{\alpha_{0.3}^{1/7} M_7^{5/21}} \tag{37}$$

$$c_s \sim 10^7 {\rm cm/s} \frac{(M_{\rm bh}/10M_\odot)^{5/21} M_7^{11/42}}{(\epsilon/\dot{m})^{1/7} \alpha_{0.3}^{1/7} (r/0.1{\rm pc})^{1/2}}, \tag{38}$$

$$\frac{N_{rmbh} \dot{M}_{\rm bh}}{\dot{M}} \sim 0.08 \frac{(M_{\rm bh}/M_\odot)^{5/21}}{\alpha_{0.3}^{1/7} \epsilon^{1/7} \dot{m}^{6/7} M_7^{5/21}}. \tag{39}$$

The last quantity is the characteristic fraction of the disc mass which gets converted into embedded black holes.

4.2 The "Eddington" model

The Eddington acretion rate for the embedded black hole is

$$\dot{M}_{\rm bh} = \frac{4\pi G M_{\rm bh}}{\epsilon_a \kappa_0 c}, \tag{40}$$

where ϵ_a is the radiative efficiency of an accretion flow. We analyze the structure of the disc by repeating the steps outlined in the previous section, and by adopting the upper limit $(M/M_{\rm bh})^{1/3}$ for the number of the black holes embedded in the disc. We find

$$Q \sim 3 * 10^5 \alpha_{0.3} (M_{\rm bh}/M)^2 (\kappa/\kappa_0)^3, \tag{41}$$

and

$$h/r \sim 2(\kappa/\kappa_0)(M_{\rm bh}/M)^{2/3}. \tag{42}$$

However an embedded black hole will open a gap if its mass is bigger than

$$M_{\rm bhgap} \simeq \sqrt{40\alpha} (h/r)^{2.5} M, \tag{43}$$

see Eqs. (4) of Lin and Papaloizou (1986). Using Eq. (42), we see that

$$M_{\rm bhgap}/M_{\rm bh} \sim 20 (M_{\rm bh}/M)^{2/3} \ll 1. \tag{44}$$

The inequality above holds unless we allow the individual masses of the embedded holes be of order of tens of thousands solar masses. Therefore it is impossible to maintain a stable disc by using the feedback from Eddington-limited black holes: either these holes are not massive enough to provide sufficient feedback, or they are so massive that they open gaps in and become decoupled from the disc. An unsupported disc would fragment completely and the fuel supply through the disc to the central engine would be stopped.

5 MERGER OF THE CENTRAL BLACK HOLE AND THE DISC-BORN BLACK HOLE.

It is likely that the newly-born black hole in the disc inspirals towards the central black hole. We imagine that the stellar-mass black hole is embedded into a massive accretion disc which forms due to continuing infall of gas from the galactic bulge, after the black hole is born. If the black hole opens a gap in the disc, it will move towards the central black hole

¶ We envisage that each hole powers a minijet which carries a significant fraction of the accretion energy.

together with the disc (type-II migration; Gould and Rix 2000, and Armitage and Natarajan 2001). The timescale for such inspiral is the accretion timescale,

$$t_{\rm inspiral} \sim 10^6 {\rm yr} \frac{M_7^{-1/2}}{\alpha_{0.1}} \left(\frac{0.1{\rm pc}}{r}\right)^{-3/2} \left(\frac{r}{30h}\right)^2. \quad (45)$$

If on the other hand, the black hole is not massive enough to open the gap, it will migrate inwards by exciting density waves in the disc (type-I migration). The speed of this inward drift is given by

$$v_{\rm in} \sim 2\beta \frac{G^2 M_{\rm bh} \Sigma h}{c_s^3 r}; \quad (46)$$

cf. Eqs. (9), (B4) and (B5) of Rafikov (2002), and Ward (1986). Here β is a numerical factor of order 5 for $Q \gg 1$, and can be significantly larger for $Q \sim 1$. For a self-gravitating disc with $Q \sim 1$, we find

$$t_{\rm inspiral} \sim 10^7 {\rm yr} \frac{5}{\beta} \frac{30h}{r} \frac{10 M_\odot}{M_{\rm bh}} \left(\frac{r}{0.1{\rm pc}}\right)^{1.5} M_7^{0.5}. \quad (47)$$

What determines whether the gap is open or not is whether a stellar-mass black hole has time to accrete a few thousand solar masses of gas, which would put it above the gap-opening threshold for the typical disc parameters. The Eddington-limited accretion occurs on a timescale of $\sim 10^8$yr, much longer than the characteristic type-I inspiral time. Thus, in this case the black holes don't accrete much on their way in. On the other hand, the Bondi-Hoyle formula predicts the mass e-folding timescale of a few hundred years. Thus, if the black hole is allowed to accrete at the Bondi-Hoyle rate, its mass increases rapidly until it opens a gap in the disc. Then, the inspiral proceeds via type-II migration.

From Eqs. (45) and (47) we see that the embedded black hole experiencing type-I or type-II migration would merge with the central black hole on the timescale 10^6—10^7 years, shorter than the typical timescale of an AGN activity. Thus it is plausible that the daughter disc-born black hole is brought towards the parent central black hole; the mass of the daughter black hole might grow significantly on the way in. Gravitational radiation will eventually become the dominant mechanism driving the inspiral, and the final merger will produce copious amount of gravitational waves. In the next subsection we show that these waves are detectable by LISA for a broad range of the black hole masses and the disc accretion rate.

5.1 Influence of the accretion disc on the inspiral signal as seen by LISA

It is realistic to expect that LISA would follow the last year of the inspiral of the disc-born black hole into the central black hole. Generally, one must develop a set of templates which densely span the parameter space of possible inspiral signals. In order for the final inspiral to be detectable, one of the templates must follow the signal with the phaseshift between the two not exceeding a fraction of a cycle. Therefore, if the drag from the disc will alter the waveform by a fraction of a cycle over the signal integration time (e.g., 1 year), detection of the signal with high signal-to-noise ratio will become problematic. Below we address the influence of the accretion disc on the final inpiral waveform.

The issue of gas-drag influence on the LISA signal was first addressed by Narayan (2000); see also Chakrabarti (1996). Narayan's analysis is directly applicable to low-luminocity non-radiative quasi-spherical accretion flows, which might exist around supermassive black holes when the accretion rate is < 0.01 of the Eddington limit. Narayan concluded that non-radiative flows will not have any observable influence on the gravitational-wave signals seen by LISA. Below we extend Narayan's analysis to the case of radiative disc-like flows with high accretion rate, which are likely to be present in high-luminocity AGNs.

Consider a non-rotating central black hole of mass $M_6 \times 10^6 M_\odot$, accreting at a significant fraction $\dot{m}$ of the Eddington rate, $\dot{M} = \dot{m}\dot{M}_{\rm edd}$. The accretion disc in the region of interest ($< 10R_S$, where R_s is the Schwartzchild radius of the central black hole) is radiation-pressure dominated, and the opacity $\kappa = 0.4{\rm cm}^2/{\rm g}$ is due to the Thompson scattering. By following the standard thin-disc theory‖ (Shakura and Sunyaev, 1973), we get

$$\Sigma(r) = \frac{64\pi c^2}{27\alpha\Omega\kappa^2 \dot{M}} \sim 4{\rm g/cm}^2 \frac{\epsilon}{\alpha\dot{m}} \left(\frac{r}{r_s}\right)^{3/2}, \quad (48)$$

where ϵ is the efficiency with which the accreted mass converts into radiation, and r_s is the Schwartzchild radius of the central black hole; and

$$c_s = \frac{3\dot{M}\Omega\kappa}{8\pi c}. \quad (49)$$

The disc black hole in orbit around the central black hole excites density waves in the disc (Goldreich and Tremaine, 1980); these waves carry angular momentum flux

$$F_0 \sim (G M_{\rm bh})^2 \frac{\Sigma r \Omega}{c_s^3}; \quad (50)$$

here, as before, $M_{\rm bh}$ is the mass of the orbiting disc black hole. Ward (1987) has argued that the torque acting on the orbiting body is $dL_{\rm dw}/dt \sim (h/r)F_0$. We can compute the characteristic timescale for the orbit evolution due to the density-waves torque:

$$t_{\rm dw} = \frac{L}{dL_{\rm dw}/dt} = \frac{1}{\Omega} \frac{M}{M_{\rm bh}} \frac{M}{\Sigma r^2} \left(\frac{h}{r}\right)^2. \quad (51)$$

One must compare $t_{\rm dw}$ to the timescale $t_{\rm gw}$ of orbital evolution due to gravitational-radiation-reaction tourque:

$$t_{\rm gw} = 8 t_{\rm m} = \frac{5}{8} \frac{c r_s^2}{G M_{\rm bh}} \left(\frac{r}{r_s}\right)^4. \quad (52)$$

Here, t_m is the time left before the disc and central holes merge, i.e. the integration time for the LISA signal. Optimistically we could expect to follow the LISA signal to 0.1 of a cycle (Thorne 1999). Therefore, if $q = 10 n_{\rm m} t_{\rm gw}/t_{\rm dw}$ is less than unity, the disc drag does not impact detection of the final inspiral; cf. Eq. (16) of Narayan (2000). Here $n_{\rm m} = \Omega t_{\rm gw}/(5\pi)$ is the number of cycles the disc black hole will make before merging with the central black hole. Using Eqs. (48), (49), (51), and (52), we get

$$q \simeq 2 \times 10^{-7} \frac{\epsilon_{0.1}^3}{\dot{m}^3 \alpha_{0.1}} \frac{(M_{\rm bh}/10M_\odot)^{13/8}}{M_6^{13/4}} t_{m,yr}^{21/8}, \quad (53)$$

‖ The disc is no longer thin close to the central black hole, however our estimates of the disc structure should be correct to an order of magnitude.

where $\epsilon = 0.1\epsilon_{0.1}$, $\alpha = 0.1\alpha_{0.1}$, and $t_m = 1\text{yr} * t_{m,yr}$. We see that for a large range of parameters $q < 1$, and the disc drag does not influence the inpiral signal. However, note that q is a very sensitive function of $\dot{m}$ and $M_{\rm bh}$. For instance, the disc will influence significantly an inspiralling $100 M_\odot$ black hole when the accretion rate is down to a few percent of the eddington limit. One then needs to reduce the influence of the disc on the LISA signal by choosing to observe the smaller portion of the final inspiral, i.e. by choosing a smaller integration time t_m.

There is another important source of drag experienced by the inpiralling black hole; it was first analyzed by Chakrabarti (1993, 1996). Generally, there is a radial pressure gradient in an accretion disc; this pressure gradient makes the asimuthal velocity of the disc gas slightly different from a velocity of the test particle on a circular orbit at the same radius. Therefore the inspiralling black hole will experience a head wind from a gas in the accretion disc; by accreting gas from the disc the black hole will experience the braking torque which will make it lose it's specific angular momentum. This tourque is given by

$$\tau_{\rm wind} = \dot{M}_{\rm bh} \Delta v r, \tag{54}$$

where $\dot{M}_{\rm bh}$ is rate of accretion onto the inspiraling black hole from the disc, and $\Delta v \sim c_s^2 / v_o$ is the speed of the headwind experienced by the black hole moving with the orbital speed v_o. We assume that the inspiraling hole accretes with the Bondi-Hoyle rate,

$$\dot{M}_{\rm bh} \simeq \pi\rho (G M_{\rm bh})^2 / c_s^3, \tag{55}$$

where $\rho = \Sigma/h = \Sigma\Omega/c_s$ is the density of the ambient disc gas. By using the last equation in Eq. (54), we obtain the expression for the tourque $\tau_{\rm wind}$, which turns out to be the same as the tourque from the density waves, up to the numerical factor between 1 and 10. We see therefore that inclusion of Chakrabarti's "accretion" torque is important for the detailed analysis, but does not qualitatively change our conclusions.

Can the orbiting black hole open a gap in the disc during it's final inspiral? In order to overcome the viscous stresses which oppose opening the gap, the mass of the orbiting black hole should be greater than the threshold value given by Eq. (43). The radius $r_{\rm in}$ from which the inspiral begins is given by

$$r_{\rm in} = 4 (M_{\rm bh}/10 M_\odot)^{1/4} M_6^{-1/2} t_{m,yr}^{1/4} r_s, \tag{56}$$

and the scaleheight of the disc is radius-independent in the radiation-dominated inner region (this is true only if you treat the central black hole as a Newtonian object):

$$h = \frac{3\dot{m}}{4\epsilon} r_s. \tag{57}$$

The gap will be open in the disc, therefore, if the mass of the inspiraling black hole exceeds

$$m_{\rm gap} \sim \dot{m}^{20/13} M_6^{-2/13} 10^4 M_\odot. \tag{58}$$

During the final year of merger of the $10 M_\odot$ disc-born black hole and the $10^6 M_{\rm odot}$ central black hole, the gap will be open if the accretion rate onto the central black hole is about five percent of the Eddington limit. Such gap-opening merger can result in sudden changes in the AGN luminosity and produce an optical counterpart to the gravitational-wave signal.

5.2 The merger event rate as seen by LISA

The details of black-hole formation and evolution in the disc are uncertain, and it seems impossible to make a reliable estimate of an event rate for LISA from this channel of black-hole mergers. Nonetheless, from Eq (39) we see that in order for an extended accretion disc in a bright AGN to be stable, a significant mass fraction of the material acreted by the cental black hole must go into the stellar-mass black holes embedded in the disc. It is worth working through a simple example to show that the merger of disc-born holes with the central hole may be an important source for LISA.

Studies of integrated light coming from Galactic Nuclei show that the supermassive black holes acquire significant part and perhaps almost all of their mass via accretion of gas; see, e.g., Yu and Tremaine (2002) and references therein. Assume, for the sake of our example, that a mass fraction η of this gas is converted into $100 M_\odot$ black holes on the way in, and that all of these black holes eventually merge with the central black hole. LISA can detect such mergers to $z = 1$ if the central black hole is between 10^5 and 10^7 solar masses (one needs to assume that the central black hole is rapidly rotating at the high-mass end of this range). The mass density of such black holes in the local universe is $\sim 10^5 M_\odot/\text{Mpc}^3$ (Salucci et. al. 1999). We can estimate, using Figure 3 of Pei et. al. (1995), that about 10 percent of integrated radiation from Galactic Nuclei comes from redshifts accessible to LISA, $z < 1$. This implies that supermassive black holes acquired 10 percent of their mass at $z < 1$. We shall therefore assume that 10 percent of the mass of the black holes in LISA mass range was accreted at $z < 1$. This implies that there were $\sim 100\eta = \eta_{0.01}$ mergers per mpc^3 which are potentially detectable by LISA. When we multiply this by volume out to $z = 1$ and divide it by the Hubble time, we get an estimate of the LISA event rate from such mergers,

$$dN/dt_{\rm toymodel} \sim 10\eta_{0.01}/\text{yr}. \tag{59}$$

The estimate above should be treated as an illustration of importance for our channel of the mergers, rather than as a concrete prediction for LISA.

Currently, it is not known how to compute well the gravitational waveform produced by an inpiral with an arbitrary eccentricity and inclination relative to the central black hole. This might pose a great challenge to the LISA data analysis. Indeed, in the currently popular astrophysical scenario, the stellar-mass black hole gets captured on a highly eccentric orbit with arbitrary inclination relative to the central black hole (Siggudrson and Rees, 1997). By contrast, in our scenario the inpiral occurs in the equatorial plane, and the signal is well understood (Hughes 2001 and references therein). Hence the template for detection is readily available, and the merger channel we consider has a clear observational signature which distinguishes it from other channels. It is an open question whether the inpiraling hole can acquire high eccentricity by ineracting with the disc or with other orbiting masses (Goldreich and Sari 2003, Chiang et. al. 2002); it seems likely that at least in some cases interaction with the disc will act to circularize the orbit. Finn and Thorne (2000) have performed a detailed census of parameter space for circular-orbit inspirals as seen by LISA.

6 CONCLUSIONS

In this paper, we rely on numerical simulations by Gammie (2001) to develop a formalism for self-gravitating thin discs which are gaining mass by continuous infall. We present a way to calculate the critical temperature, surface density, and scaleheigh of the disc just prior to fragmentation. Our formalism naturally includes both optically thin and optically thick discs.

We then speculate on the outcome of the nonlinear physical processes which follow fragmentation: accretion and merger of smaller fragments into bigger ones. We find an upper bound on the mass of final, merged fragments, and we give a plausibility argument that some fragments will indeed reach this upper bound. We thus predict that very massive stars of tens or even hundreds of solar masses will be produced in self-gravitating discs around supermassive black holes. The end product of fast evolution of these massive stars will be stellar-mass black holes.

Finally, we make an argument that the disc-born black holes in AGNs find a way to merge with central black holes. We consider a purely toy-model example of what the rate of such mergers might be, as seen by LISA; we illustrate this rate might be high enough to be interesting for future gravitational-wave (GW) observations. The GW signal from this merger channel is distinct from that of other channels, and can be readily modeled using our current theoretical understanding of the final stages of the inspiral driven by gravitational-radiation reaction. We show that for a broad range of accretion rates and black-hole masses the drag from accretion disc will not be large enough to pollute the signal and make inspiral template invalid. In some cases, the inspiralling hole will open a gap in the accretion disc close to the central black hole, thus producing a possible optical counterpart for the gravitational-wave burst generated by the merger.

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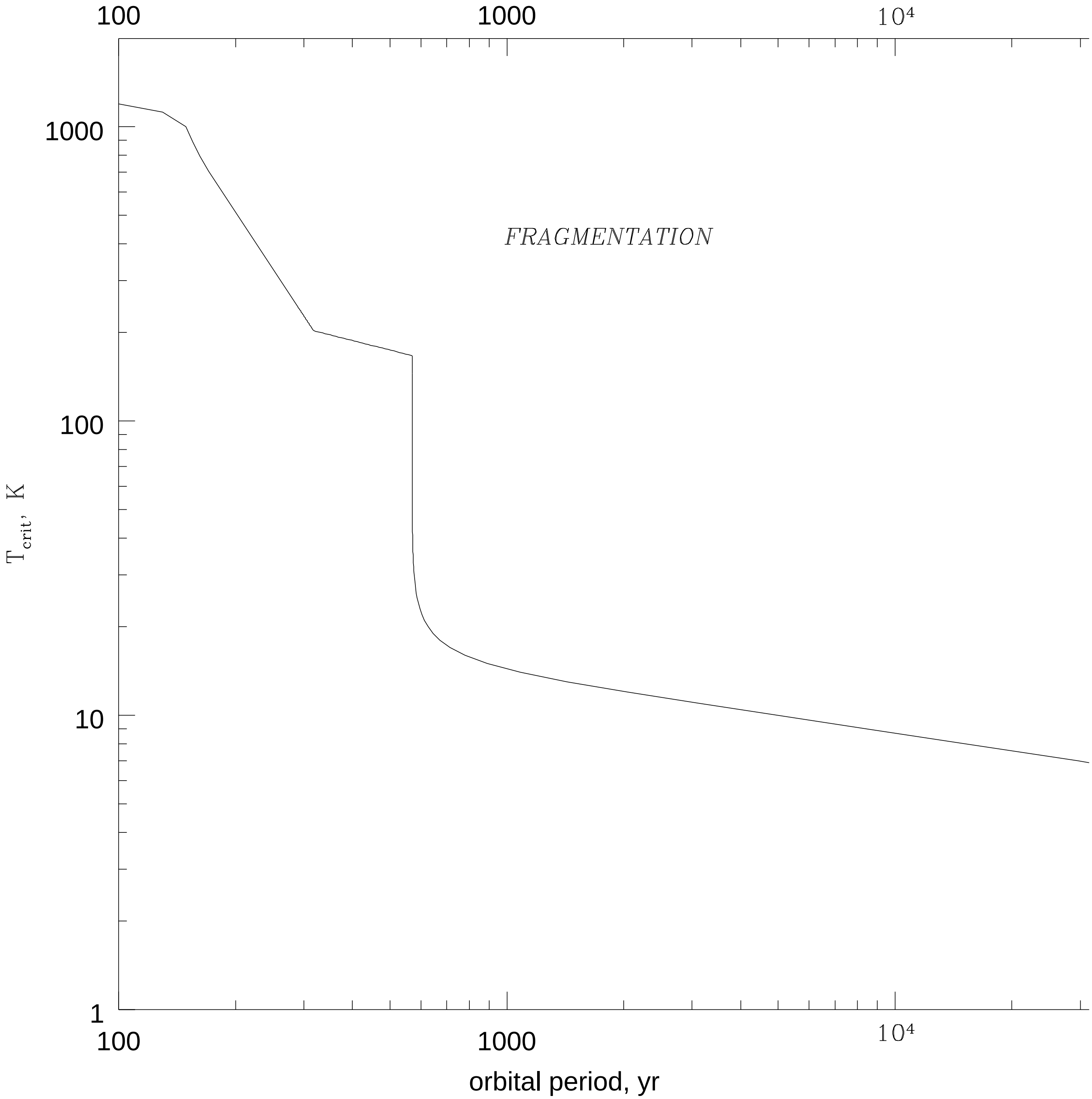
Figure 1. The temparature of the critically fragmenting disc as a function of the orbital period.

Figure 2. The surface density of the critically fragmenting disc as a function of the orbital period.

Figure 3. The Toomre mass of the critically fragmenting disc as a function of the orbital period.

Figure 4. The isolation and gap opening masses plotted as a function of radius for the critically fragmenting disc without external sources of heating. The black hole mass is taken to be $3 \times 10^6 M_\odot$.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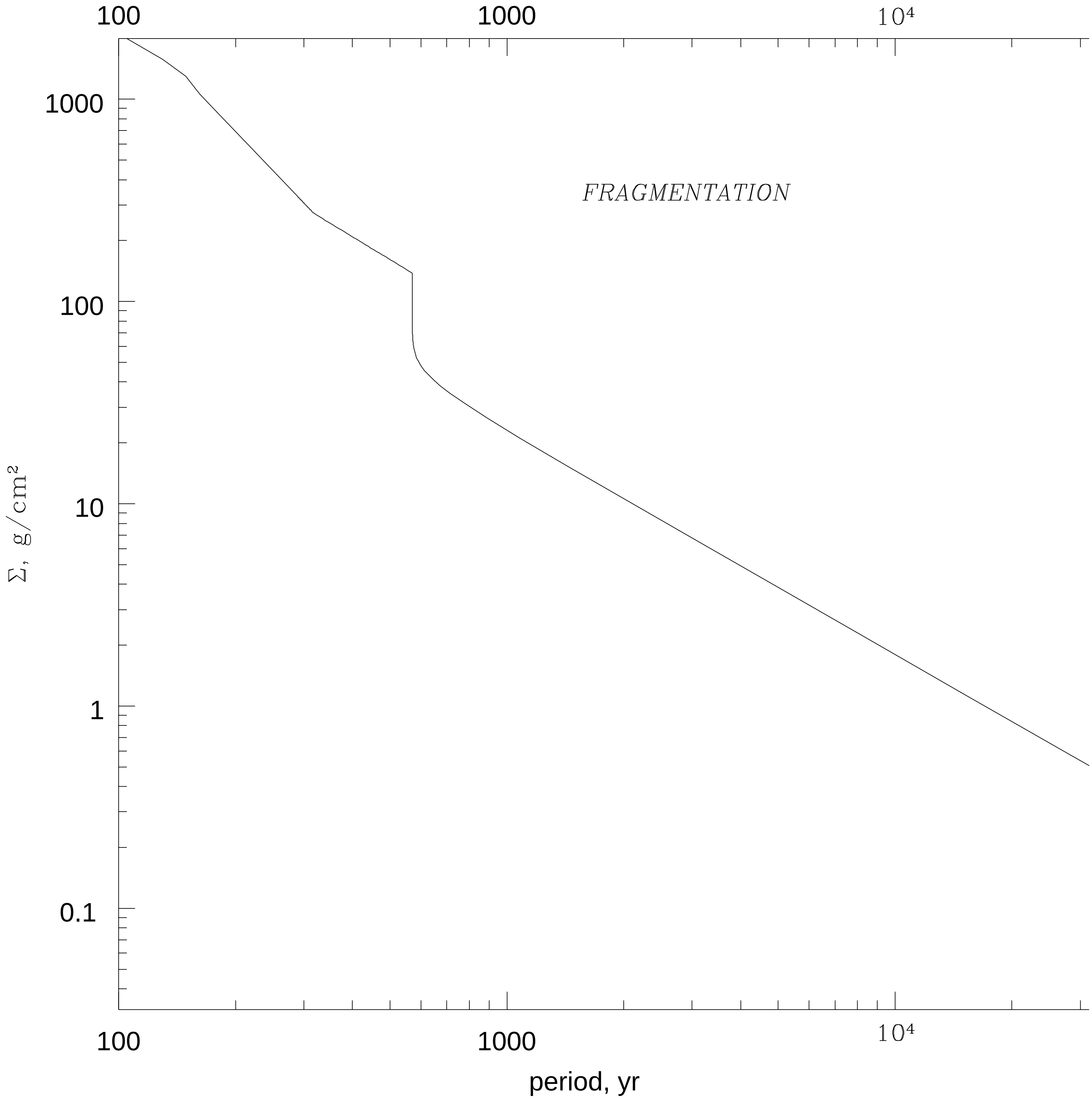
1000
10^4
1000
100
10
1
FRAGMENTATION
T_{crit}, K
100
1000
10^4
orbital period, yr



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10
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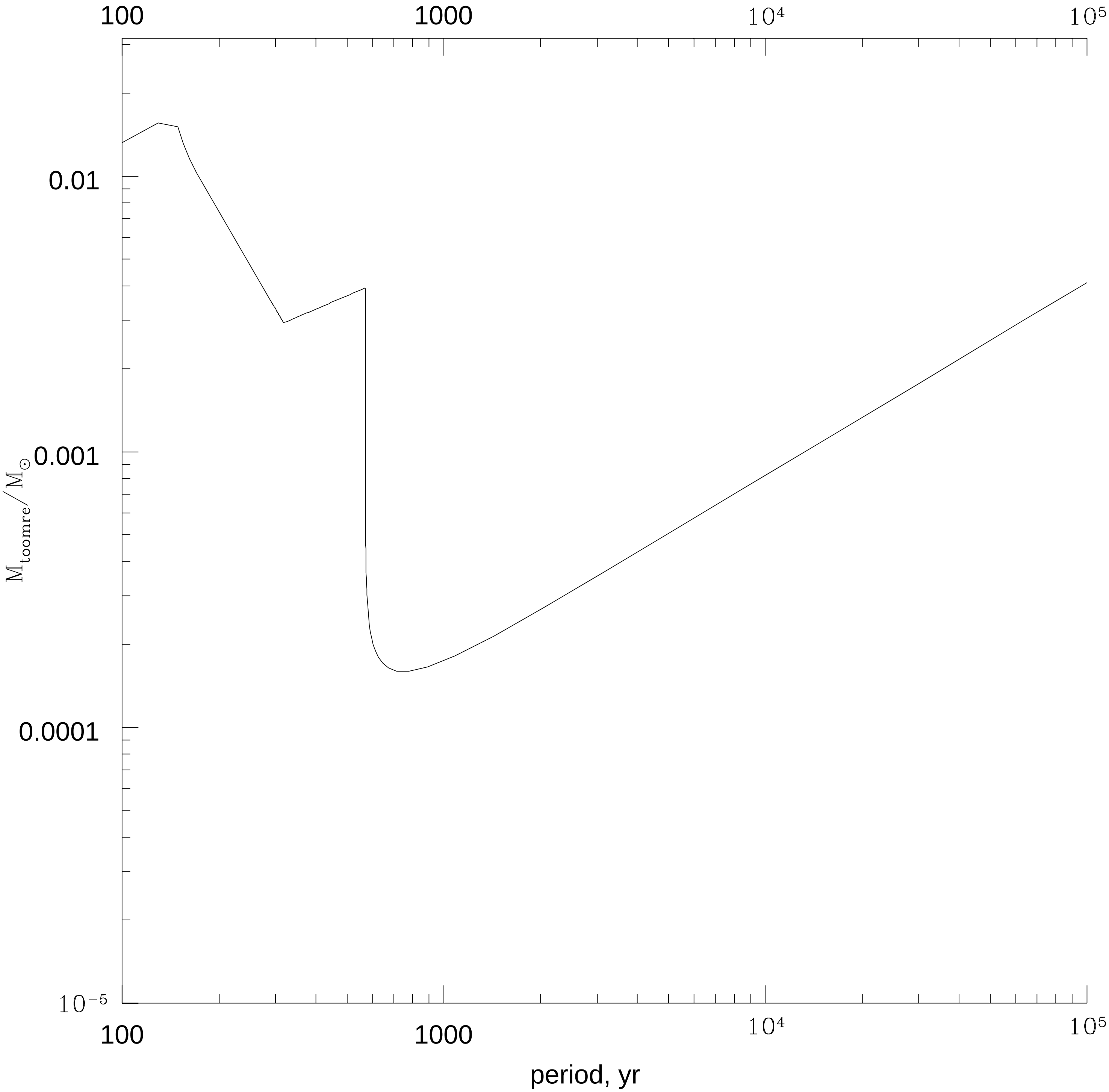
0.1
FRAGMENTATION
Σ, g/cm^2
period, yr



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10^4
10^5
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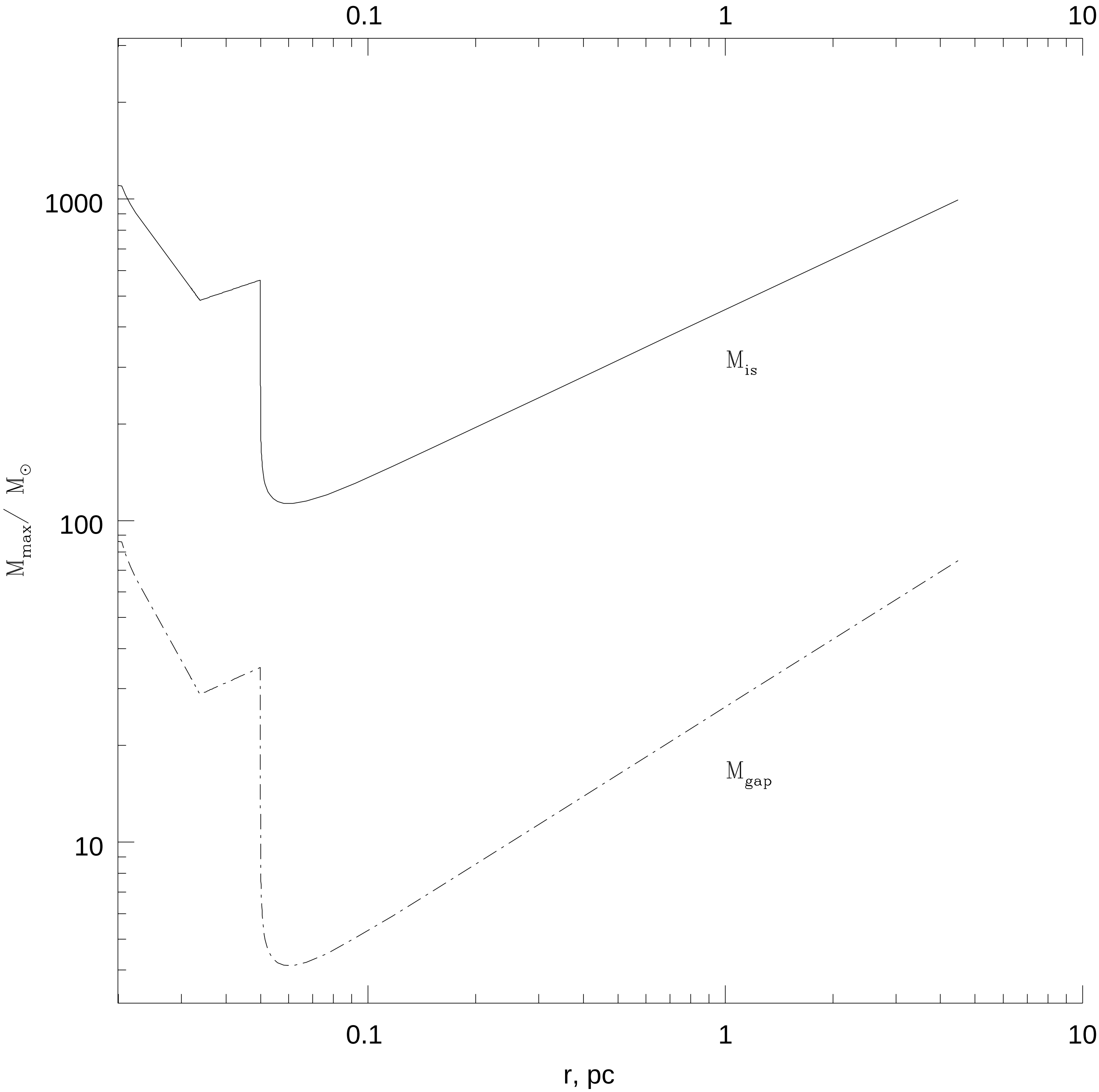
0.001
0.0001
10^{-5}
$M_{toomre}/M_\odot$
period, yr



0.1
1
10
1000
100
10
$M_{max}/M_\odot$
M_{is}
M_{gap}
r, pc